Physics Factsheet
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A.C. (Alternating Current)

Before dealing with alternating current (a.c.), it's probably worth consolidating what we understand about direct current (d.c.).

Any electrical current is a flow of charged particles, and in most cases this will be a flow of free electrons in a conductor such as copper.

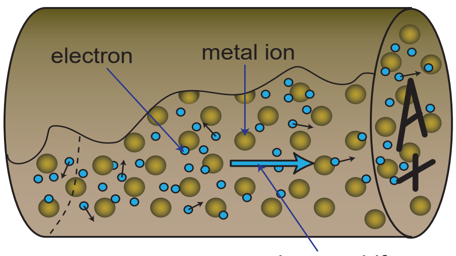
Each electron has a charge of -1.6×10^{-19} Coulombs and the current itself (represented by the letter I) is defined as the rate of flow of charge, Q, and can be expressed:

$$I = \frac{\Delta Q}{\Delta t}$$

The electrons themselves are made to flow when a potential difference is set up across the conductors in the circuit, with one end of the circuit becoming positively-charged compared to the other end. This potential difference 'V' attracts the electrons (having negative charge) to the positive end of the circuit and repels them from the negative end of the circuit. A battery or cell provides a constant, steady potential difference and therefore a constant, steady current. This current always flows around the circuit in the same direction. This is what direct current is.

You could visualise current in a conductor as rows and rows of identical, spherical metal ions, regularly lined up within the metal, and tiny specks of charge streaming through the gaps between these spheres.

Figure 1



Needless to say, a greater potential difference will cause a greater flow or current, but the metal ions get in the way of this flow and cause resistance.

The resistance of a conductor is defined as the potential difference needed to cause 1 ampere (1 Amp) of current to flow, represented by the equation below (Ohm's Law):

$$R = \frac{V}{I}$$

Finally, by moving round the circuit the electrons can deliver energy. The rate at which energy is transferred is the power, which can be calculated from the potential difference applied to a particular part of the circuit and the current flowing through it:

$P = I \times V$

These three equations, and the model used to visualise how current flows will be useful in understanding alternating current.

Alternating Current

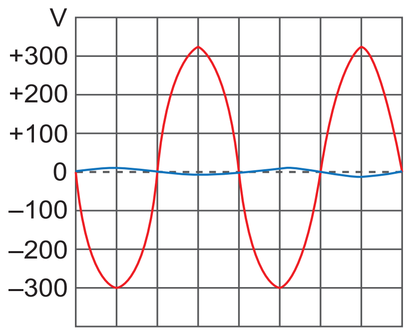
In a dynamo or generator or transformer, the potential difference (electromotive force or e.m.f.) is caused by a coil of wire experiencing a constantly-changing magnetic field, and this e.m.f. varies in a predictable, sinusoidal way.

For a supply with peak potential difference V_{peak} and of frequency f, this means the potential difference at any one instant can be expressed as:

$$Vt = V_{peak} \times \text{Sin}(2\pi f\, t)$$

> ***Exam Hint:*** *The peak potential difference is measured to the peak from zero, so in the graph below is just over 300V (approximately 310V). The peak-to-peak potential difference is simply double that, since the curve is symmetrical i.e. the peak-to-peak potential difference is 620V.*

Figure 2



As a consequence, the electrons in the circuit are repeatedly pulled to and fro within the conductors – their direction and speed is constantly changing or alternating, hence the name.

> ***Quick Question 1:*** *Does Ohm's Law still apply in this situation?*
>
> ***Answer:*** *Yes – the flow of electrons is still directly due to the potential difference and the resistance they experience in colliding with the metal ions. Therefore, the current, I, still equals V/R at any, and every, instance.*
> *So, since the potential difference at an instance, t, is given as:*
> $Vt = V_{peak} \times Sin(2\pi f\, t)$
> *And I = V/R*
> *The current at an instance 't' is given as*
> $It = I_{peak} \times Sin(2\pi f\, t)$

Quick Question 2: *An a.c. supply is rated as 200V peak p.d and 25Hz, and is connected to a circuit of resistance 2500Ω. Calculate the peak current through the circuit, and the current flowing 0.025s into a cycle.*

Answer: *Peak current:*

$I_{peak} = V_{peak}/R = 200/2500 = 0.08A$ or 80mA

At 0.025s into a cycle:

$It = I_{peak} \times Sin(2\pi f\ t)$

$= 0.08 \times Sin(2\pi \times 25 \times 0.025)$

$= 0.08 \times -0.707$

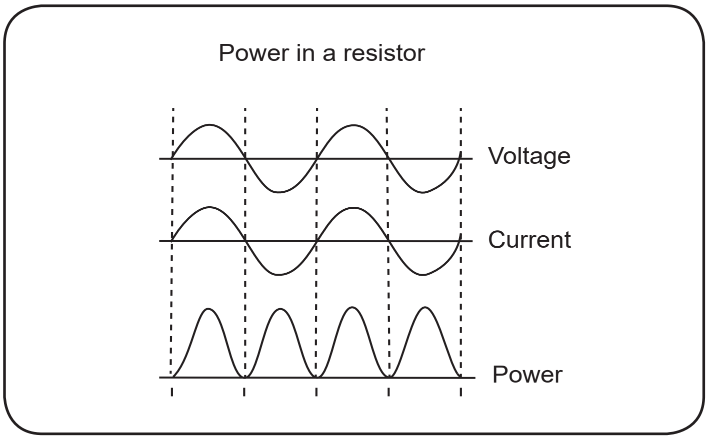
$= -0.0566A$ or $-56.6mA$

However, the upshot of this is that whilst the electrons do transfer energy within the circuit, they don't actually experience any net movement – no sooner has the rise and fall of the positive potential difference pulled them forwards than the equivalent negative potential difference pulls them back to where they started from!

This is awkward, because strictly speaking the average current is therefore zero and the average potential difference is zero. Every positive potential difference on the curve has an equal, equivalent negative potential difference that cancels it out half a cycle later. Things were so much simpler with d.c.!

Ideally, we need a way of determining the effective d.c. potential difference and current that our a.c. supply is equivalent to, in terms of the energy or power it transfers. In fact, considering the power being transferred is a good approach as it is the one thing that both types of current have in common - they both transfer power, but whilst d.c. transfers power at a constant, steady rate, a.c. transfers the power in rapid bursts.

Figure 3



Because the power being transferred is always positive, we can determine a mean value without the second part of each cycle cancelling out the first half. As shown in the diagram above, whilst the current and potential difference swing from positive to negative in a sinusoidal fashion, the power transferred at any one instance is always positive.

Quick Question 3: *If the potential difference and current are varying sinusoidally (as a sine-wave), can you describe how the power transferred is varying? And can you back that up mathematically?*

Hint: *It is not a sine-wave as there are no negative values.*

Answer: *The power varies as a Sin^2 curve.*

The reason for this is that power = current × potential difference, so if current and potential difference each vary as a sine-curve and are in-phase with each other (check your 'waves' and 'SHM' understanding if you need to check what that phrase means!) then the power transfer will be a sine curve × a sine curve, i.e a Sin^2 curve.

More rigorously:

We have seen how the current at any instant can be determined as:

$I_t = I_{peak} \times Sin(2\pi f\ t)$

And the potential difference at this instant is similarly:

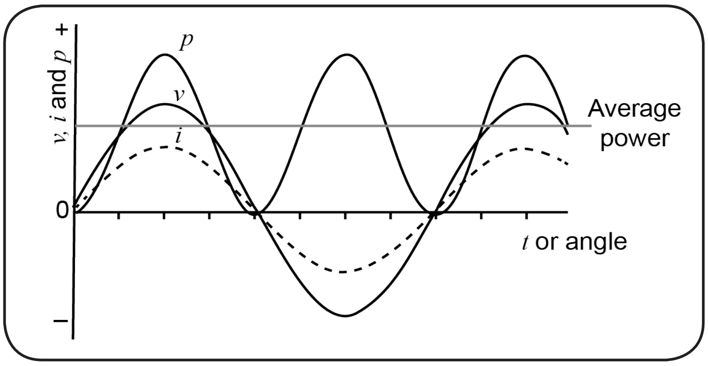
$V_t = V_{peak} \times Sin(2\pi f\ t)$

Since power = current × potential difference, the power being transferred at an instant 't' can be expressed as:

$P_t = I_t \times V_t = I_{peak} \times Sin(2\pi f\ t) \times V_{peak} \times Sin(2\pi f\ t) = I_{peak} \times V_{peak} \times Sin^2(2\pi f\ t)$

The beauty of this - as can be seen from the diagram below - is that because a Sin^2 curve is symmetrical top and bottom, the average value is exactly halfway from zero to peak.

Figure 4



The mean power transfer is half the peak power transfer i.e. exactly half of $I_{peak} \times V_{peak}$.

Exam Hint: *Once you realise that the energy transfer is the area under the curve you should spot that the energy above the mean line (the 'surplus' energy) is exactly the same shape as the gaps below the mean line (the 'missing' energy). Imagine shading in the area under the curve then comparing the shading of the peaks above the mean line with the dips of missing energy below the mean line.*

This gives us a neat way of determining the effective (d.c. equivalent) potential difference and current that an a.c. supply provides.

The effective power:

$P_{eff} = ½ \times P_{peak} = ½ \times I_{peak} \times V_{peak}$

Power also:

$I^2 \times R$

So, the effective current:

$I^2_{eff} = P_{eff}/R$

$= (½ \times I_{peak} \times V_{peak})/R$

$= ½ \times I_{peak} \times I_{peak}$ (because V/R = I)

$= ½ \times I^2_{peak}$

So since $I^2_{eff} = ½ \times I^2_{peak}$ by finding the square-root of each side we can show that:

$$I_{eff} = \frac{1}{\sqrt{2}} \times I_{peak}$$

Essentially what we have done is determined the square-root of the mean of the current².

This is abbreviated to root-mean-square (r.m.s.) and it represents the d.c. equivalent of current being delivered by the a.c. supply.

Quick Question 4: *Can you show that* $V_{rms} = \frac{1}{\sqrt{2}} \times V_{peak}$

Answer:

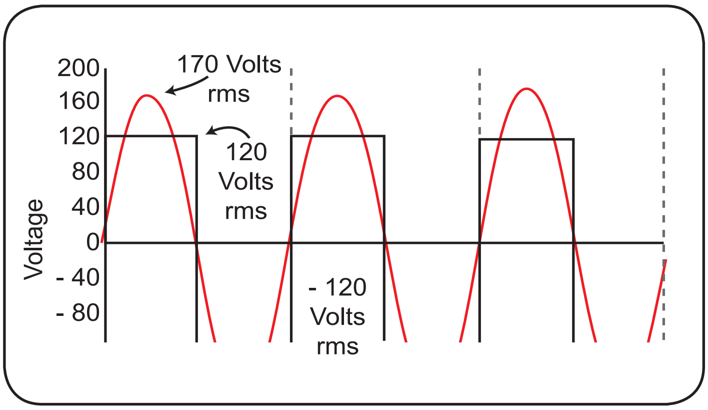
Since $V = I \times R$, $V_{rms} = I_{rms} \times R = \frac{1}{\sqrt{2}} \times I_{peak} = \frac{1}{\sqrt{2}} \times V_{peak}$

Quick Question 5: *An a.c. supply is rated as 170V peak p.d. and is connected to a circuit of resistance 200Ω. Calculate the r.m.s. potential difference and current through the circuit.*

Answer: $V_{rms} = \frac{1}{\sqrt{2}} \times V_{peak} = \frac{1}{\sqrt{2}} \times 170 = 120.2\ V$

$I_{rms} = V_{rms}/R = 120.2/200 = 0.60A$ *or 600mA*

Figure 5

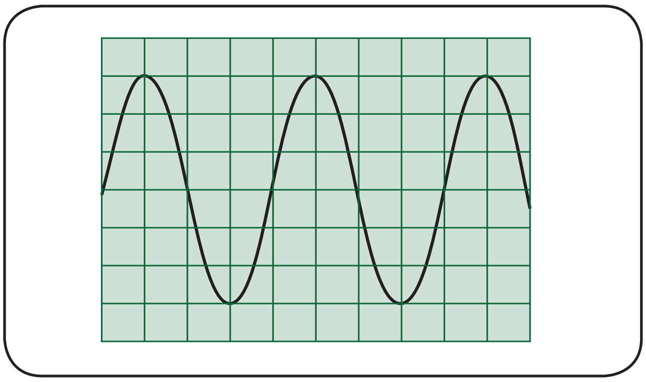


Practical Aspects

Although there are a.c. voltmeters and ammeters available, it is more common to use an oscilloscope to display an a.c. potential difference, and use this to determine the peak p.d. and the time-period for the trace, in order to calculate the r.m.s. potential difference of the supply and its frequency.

Exam Hint: *To reduce uncertainty, alter the voltage sensitivity and time-base settings on the oscilloscope so that the pattern fills at least half the screen vertically and shows no more than 2 or 3 cycles. Also check the trace gives a horizontal line exactly on the centre axis when the supply is disconnected, so the curve is vertically symmetrical on the screen.*

Figure 6



Quick Question 6: *The trace above is obtained when a supply is fed into an oscilloscope where the y-sensitivity setting is 50V/div and the timebase is set as 5ms per division. Determine the peak-to-peak p.d., the peak p.d., the r.m.s. p.d. and the frequency of the supply.*

Answer: *The trace sweeps across 6 divisions peak-to-peak, so:*

peak-to-peak p.d = 6 × 50V = 300V

peak p.d = ½ × 300V = 150V

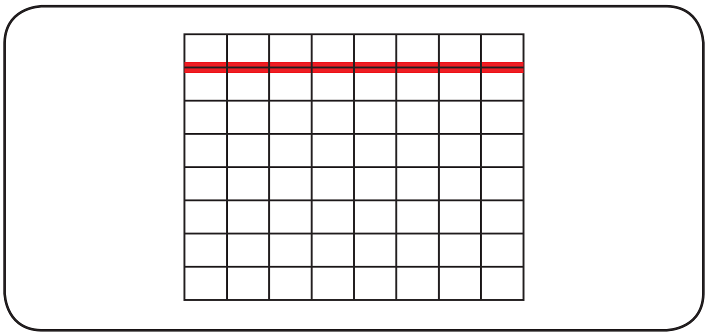
$V_{rms} = \frac{1}{\sqrt{2}} \times V_{peak} = \frac{1}{\sqrt{2}} \times 150 = 106.1V$

The trace shows one cycle as 4 divisions (or two and a half cycles for 10 divisions) = 20ms

If one cycle = 20ms = 0.02s then the frequency = 1/0.02 = 50Hz.

A d.c. supply can also be displayed on an oscilloscope screen, for example, as shown in Figure 7.

Figure 7

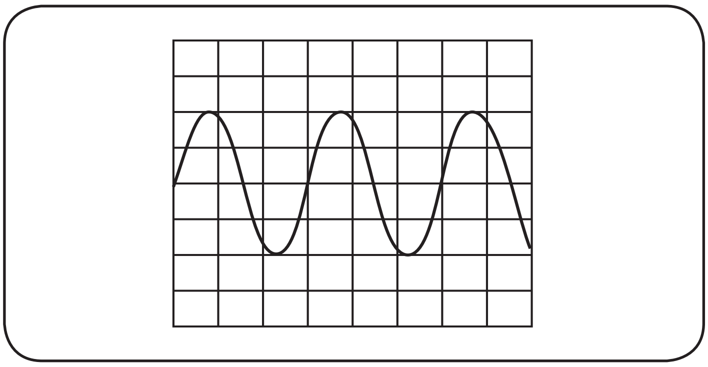


If the oscilloscope settings were unchanged, this would show a d.c. supply of 150V.

Exam Questions

1) An alternating current (a.c.) supply is connected to a resistor to form a complete circuit. The trace obtained on an oscilloscope connected across the resistor is shown.

Figure 8



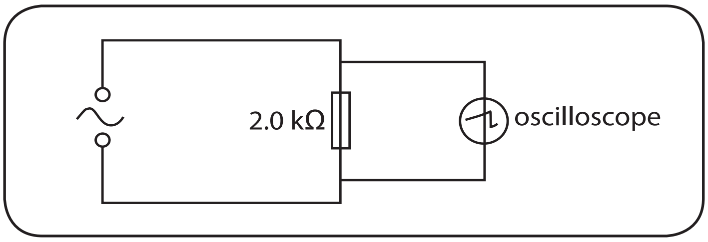
The oscilloscope settings are: Y sensitivity 5.0 V per division, time base 2.0 ms per division.

a) Determine the peak voltage of the a.c. source. (1 mark)
b) Hence calculate the r.m.s. voltage. (1 mark)
c) Determine the time period of the a.c. signal. (1 mark)
d) Hence calculate the frequency of the a.c. signal. (1 mark)

(4 marks)

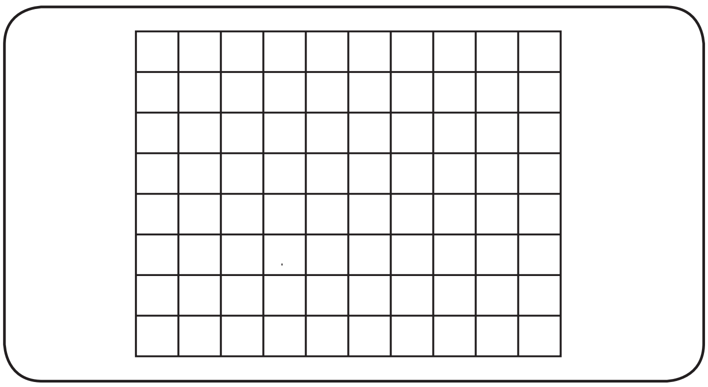
2) A sinusoidal alternating voltage source of frequency 200 Hz is connected to a resistor of resistance 2.0 kΩ and an oscilloscope, as shown below.

Figure 9



a) The r.m.s. current through the resistor is 17.3 mA. Calculate the peak voltage across the resistor. (2 marks)
b) The settings on the oscilloscope are: timebase: 1 ms per division, voltage sensitivity: 20 V per division. Draw on the grid, which represents the screen of the oscilloscope, the trace that would be seen. (4 marks)

Figure 10



Exam Question Answers

1) a) $V_0 = 10.0$ V **(1)**
b) $V_{rms} = 10/\sqrt{2} = 7.1$ (V) (allow e.c.f. from (a)) **(1)**
c) $T = 6.0$ ms **(1)**
d) $f = \frac{1}{T} = 170$ (167) Hz (allow e.c.f. from (c)) **(1)**

2) a) ($V = IR$ gives) $V_{rms} = (17.3 \times 10^{-3} \times 2 \times 10^3) = 34.6$ (V) **(1)**
$V_0 = V_{rms}\sqrt{2} = 34.6\sqrt{2} = 49$ V (1) (48.93 V) **(1)**
[or calculate I_0 (= 24.5 mA) and then V_0]
b) (use of $T = \frac{1}{f}$ gives) $T = \frac{1}{200} = 5 \times 10^{-3} = 5.0$(ms) **(1)**
Trace to show:
correct wave shape (sinusoidal) **(1)**
correct amplitude (2.5 divisions) **(1)**
correct period (5 divisions) **(1)**

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