Physics Factsheet

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Solving Problems with Energy

Ideas about energy run through pretty much every topic in physics. It is one area you can guarantee being asked questions about and being such a large area it is quite likely that you will get some more complex problem solving questions related to energy. This sheet will give you some practice at approaching these types of questions.

Know your equations !

The more easily you can bring equations to mind, the easier it will be to see what your options are in reaching a solution. If you have to look up basic equations you are less likely to see a way forward and it will take you far longer, which is no fun for anyone.

***Revision Tip:** Go through your notes and text book and write a catalogue of all the equations that have anything to do with energy.*
Arrange them by topic: general, motion, electrical, nuclear and thermal.
Use the sheet when problem solving until you become familiar with them.
Practise, practise, practise!

***Exam Hint:** Keep your working neat and orderly. Keep any quick workings to the side.*
If you get stuck in the exam don't cross anything out until you have the correct method and answer written down. Something you've written might be worth a mark, but not if you cross it out.

General Energy Equations

$P = \frac{W}{t} = \frac{E}{t}$ $\quad P = \frac{\text{work (or energy)}}{\text{time}}$

$\text{Effeciency} = \frac{\text{useful energy output}}{\text{total energy transferred}} (\times 100\%)$

Mechanical Energy Equations

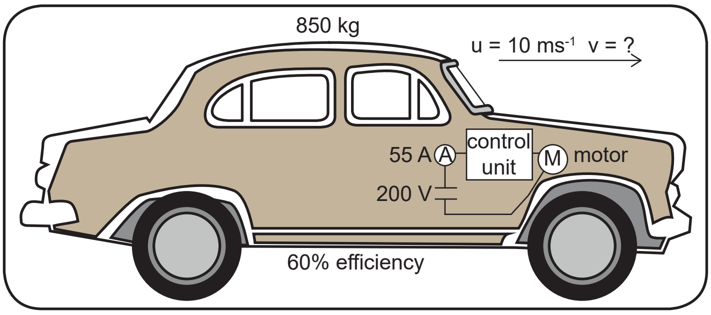
W=fs or W=fd,	work = force × distance (in direction of force)
$E = \frac{1}{2} m v^2$,	Kinetic energy = ½ mass velocity^2
$\Delta E = mg\Delta h$	Change in gravitational potential energy = mass × gravitational field strength × change in height
$P = Fv$	Power = force × velocity

Electrical energy equations

$W = QV$	Energy transferred = Charge × potential difference (p.d.)
$W = VIt$	Energy transferred = P.d. × current × time
$P = IV$	Power = current × P.d.
$P = I^2 R$	Power = current^2 × resistance
$P = \frac{V^2}{R}$	$\text{Power} = \frac{(\text{p.d})^2}{\text{Resistance}}$
$W = \frac{1}{2} Q V$	Energy stored by capacitor = ½ × Charge × P.d.
$W = C V^2$	Energy stored by capacitor = ½ × capacitance × (P.d.)^2

Question 1:

An electrically powered car of mass 850kg accelerates from rest. The 200V battery provides a current of 55A to the motor which transfers the energy with 60% efficiency. Calculate the velocity of the car after 10s. [3 marks]



Answer:

Firstly, calculate the amount of electrical energy supplied
Work done, W = P t, and P = I V
Work = I V t = 55 × 200 × 10 = 110 000 J ✓
Using an efficiency of 60%, calculate the energy transferred to kinetic energy

$\frac{110\ 000\text{J}}{100\%} \times 60\% = 66000\text{J}$ ✓

Use the kinetic energy equation $E_k = \frac{1}{2} m v^2$, rearrange for velocity, v.

$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 66000}{850}} = 12.5\text{ms}^{-1}$ ✓

Quantum, Particle and Nuclear Physics Energy Equations

$E = hf$	Energy of photon = Planck's constant × frequency
$E = \frac{hc}{\lambda}$	$\text{Energy of photon} = \frac{\text{Planck's constant} \times \text{speed of light}}{\text{wavelength}}$
$hf = \Phi + KE_{max}$	Planck's constant × frequency = work function + kinetic energy$_{max}$
$\Delta E = \Delta mc^2$	Energy (change) = (change in) mass × speed of light^2

Question 2:
An LED emits red light of frequency 4.60×10^{14} Hz when a 2.0V potential difference is applied and carries a current of 20mA. Calculate the number of photons emitted per second by the LED. [3 marks]

Answer:
First calculate the energy per photon, E = h f

(Planck's constant × frequency) = $6.63\times10^{-34} \times 4.60\times10^{14} = 3.05\times10^{-19}$J ✓

The power output of the bulb. P = I V = 20×10^{-3}A × 2.0 = 0.04 W, or J s^{-1} ✓

$$\text{no. of photons per second } s^{-1} = \frac{\text{power output}}{\text{energy of photon}}$$

$$= \frac{0.04}{3.05\times10^{-19}} = 1.31\times10^{17} \text{ photons } s^{-1}$$ ✓

Thermodynamics Energy Equations
$\Delta E = m\,c\,\Delta T$ Thermal energy change
= mass x specific heat capacity × temperature change

$\Delta E = m\,l$ Thermal energy change = mass × specific latent heat

$E = 3/2\,kT$ Energy of particle = 3/2 Boltzman constant × Temperature

Question 3:
A cyclist applies the rear brake, which presses a brake pad against the metal rim of the wheel, and brings herself rapidly to a stop from 19 m s^{-1}. The cyclist and bicycle have a total mass of 89.5 kg.

(a) Describe the transfer of energy that takes place. [1 mark]

(b) If the braking force is 180N calculate the braking distance of the bicycle. [2 marks]

(c) The brake and wheel system has mass 2.8kg and is made of an alloy with a specific heat capacity of 368 J kg^{-1} K^{-1}. If negligible energy is lost from the brake and wheel system, calculate the temperature increase of the brake system. [2 marks]

Answer:
(a) Kinetic energy to internal energy in the brakes (transferred as work done by the brakes).

(b) The work done by the brakes is equal to the initial kinetic energy of the bicycle and rider $E_k = W$, $\frac{1}{2} m v^2 = F d$, Rearrange for distance

$$d = \frac{\frac{1}{2}mv^2}{F} = \frac{\frac{1}{2}\times89.5\times19^2}{180} = 89.7\,m$$ ✓✓

(c) Kinetic energy transferred is equal to the change in internal energy (specific heat capacity). $E_k = E_T$, $\frac{1}{2} m_c v^2 = m_b c \Delta T$ (m_c is mass of cyclist and bicycle, m_b is mass of brake system) ✓
Rearrange for temperature change,

$$\Delta T = \frac{\frac{1}{2}m_c v^2}{m_b c} = \frac{\frac{1}{2}\times89.5\times19^2}{2.8\times368} = 15.7\,K$$ ✓

Energy in Fields (Gravitational and Electric)
$$E = \frac{GMm}{r}$$

$$\text{gravitational potential energy} = \frac{\text{grav.constant} \times \text{mass}_1 \times \text{mass}_2}{\text{separation}}$$

$$V_g = \frac{G\,M}{r} \qquad \text{gravitational energy} = \frac{\text{grav.constant} \times \text{mass}}{\text{distance}}$$

$$E = \frac{Qq}{4\pi\varepsilon_0 r}$$

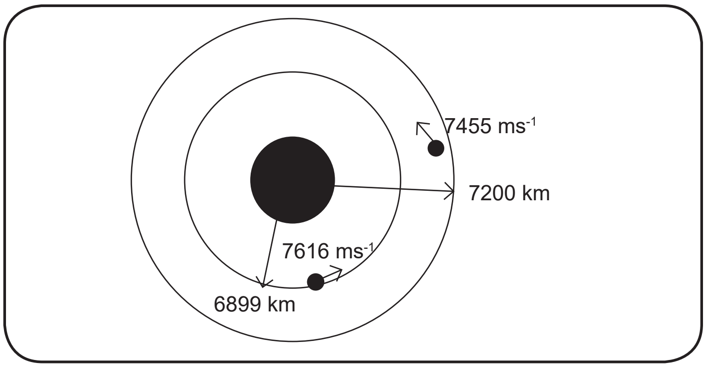
$$\text{Electric potential energy} = \frac{\text{Charge}_1 \times \text{Charge}_2}{4\pi \times \text{permativity of free space} \times \text{separation}}$$

$$V = \frac{Q}{4\pi\varepsilon_0 r} \qquad \text{Electric potential} = \frac{\text{charge}}{\text{distance}}$$

Exam Hint: *If you get a complicated question, help yourself visualise the problem and put all the information you have together in one place using a quick sketch.*

Question 4:
A space shuttle of mass 2.03×10^6 kg is in a circular orbit of radius 6 899 km around the Earth travelling at 7616 m s^{-1}. In order to place a satellite in the correct orbit, it must reach an orbit of radius 7200 km and will have to slow to a speed of 7455m s^{-1}. Calculate the net energy change involved in moving the shuttle to the higher orbit. (Mass of Earth 6.0×10^{24} kg) [4 marks]

Answer:
No diagram was supplied but a sketch will certainly help us answer this one.



Before we answer, we have to consider the energy changes that have occurred:

Kinetic Energy – as the shuttle has slowed down, this kinetic energy has decreased, that's easy.

Gravitational potential energy, $E = \frac{-G\,M\,m}{r}$

As the radius of the orbit increases the magnitude of the potential energy will decrease. However, remember that potential energy is negative and is defined as the energy required to move an object from R to infinity.

Therefore, when the magnitude of R decreases, we actually say it has got *less negative* which means it has actually *increased*.

Change in Kinetic Energy
$$\Delta E_k = \tfrac{1}{2}m(v_2)^2 - v_1^2 = \tfrac{1}{2}(2.03\times10^6[7616]^2 - 7455^2) = 2.46\times10^{12}\text{ J}$$ ✓

This energy is *lost* from the shuttle

Change in Gravitational potential energy

$$E = GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$= -6.67\times10^{-11} \times 6.0\times10^{24} \times 2.03\times10^6\left(\frac{1}{6.899\times10^6}\right) - \left(\frac{1}{7.2\times10^6}\right)$$

= -4.92×10^{12} J ✓

This energy is *gained* by the shuttle

Net energy change = 2.46×10^{12} + (-4.92×10^{12} J) = -2.46×10^{12}

The shuttle loses 2.46×10^{12} J of energy. ✓

Strain (Spring) Energy

$E = ½ F x$ Energy stored = ½ × force (load) × extension
$E = ½ k x^2$ Energy stored = ½ × spring constant × extension^2

> ***Exam Hint:*** *There are often two ways to answer energy questions. You can work out individual energies and combine them to calculate what you need to find, or you can combine the equations to make a single equation you can place all your information in.*

Question 5:

A 16g ball bearing is fired from a spring powered gun. The spring has a spring constant of 480 N m^{-1} and is compressed by 5cm. The ball bearing is fired vertically upwards. Calculate the height it will reach if air resistance is negligible. [2 marks]

Answer:

The strain potential energy will convert to kinetic energy and then to gravitational potential energy. At the highest point, there will be no kinetic. So, we can say:
Strain potential energy = gravitational potential energy

$$½ Fx^2 = mg\Delta h ✓ \quad \Delta h = \frac{½kx^2}{mg} = \frac{½480 \times 0.05^2}{0.016 \times 9.81} = 3.82 \text{ m}$$

Question 6:

A beam of electrons is accelerated to a velocity of 2.651×10^8 m s^{-1}. Calculate the stopping potential for the electrons. (rest mass of electrons 9.11×10^{-31} kg) [3 marks]

Answer:

Kinetic energy = $½mv^2 = ½ \times 9.11\times10^{-31} \times (2.651\times10^8)^2 = 3.2\times10^{-14}$ J✓

W=QV rearranged to give

$$V = \frac{W}{Q} = \frac{3.2\times10^{-14}}{1.6\times10^{-19}} = 200\,000 = 200 \text{ kV} ✓✓$$

> ***Exam Hint:*** *'Show that' questions require careful demonstration of working. You need to show how you reached an answer but you often won't get marks for the actual answer. If you are 'showing' a numerical answer you may need to round yours to get the one you are given.*

Question 7:

An isotope of Carbon-14 decays to produce a beta particle and forms Nitrogen-14. If 25% of the energy produced by the decay goes to the beta particle, show that its velocity is 1.7×10^8 m s^{-1} if relativistic effects can be ignored. [3 marks]

Carbon – 14 rest mass 2.32529×10^{-26} kg

Nitrogen – 14 rest mass 2.32526×10^{-26} kg

Beta particle (electron) rest mass 9.109×10^{-31} kg.

Answer:

Mass defect = $2.32529\times10^{-26} - (2.32526\times10^{-26} + 9.109\times10^{-31}) = 6.109\times10^{-31}$.
$E=mc^2 = 5.4981\times10^{-14}$J ✓
The kinetic energy of the beta particle:

$$25\%: \frac{5.4981\times10^{-14}}{100} \times 25 = 1.3745\times10^{-14} \text{ J}$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times1.3745\times10^{-14}}{9.109\times10^{-31}}} = 1.73732\times10^8 \text{ ms}^{-1} ✓✓$$

Question 8:

A ball is dropped from height h and hits the ground with velocity v. Show that if the ball is dropped from half the height it will reach the ground with velocity 0.7v. [2 marks]

Answer:

Kinetic energy at bottom = gravitational potential energy at the top

$½ m v^2 = m g \Delta h$

Initially:

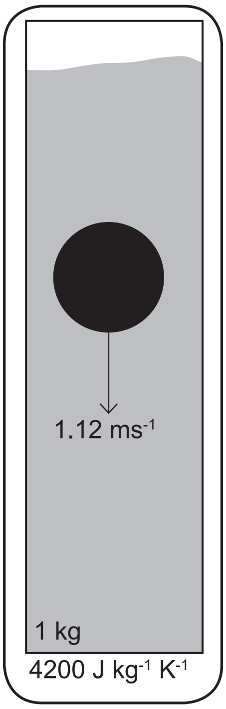
$$v = \sqrt{\frac{2mgh}{m}} = \sqrt{2gh}$$

When the ball is dropped from half the height

$$v = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{\tfrac{1}{2} \times 2gh} = \sqrt{\frac{1}{2}}\sqrt{2gh} = 0.7\,v ✓✓$$

Question 9:

A 0.300kg ball falls at a constant velocity of 1.12 m s^{-1} through 1kg of water. (specific heat capacity of water 4200 J kg^{-1} K^{-1}). Calculate the rate of temperature change on the water. [3 marks]



Answer:

Calculate the rate of energy lost by the ball (rate of change of GPE)

$$P = \frac{E}{t} = \frac{Mgh}{t} = \frac{0.3 \times 9.81 \times 1.12}{1} = 3.296\text{W} ✓✓$$

Calculate the temperature change of the water per second.

$$\Delta t = \frac{E}{mc} = \frac{3.296}{1 \times 4200} = 7.84\times10^{-4} \text{ K s}^{-1} ✓$$

Question 10:

Electrons are accelerated through a potential difference and fired at a tungsten plate to create X-rays of wavelength 10nm. Calculate the potential difference needed to create X-rays of this wavelength. [2 marks]

Answer:

Energy of the X-ray is equal to the kinetic energy of the electron.

$$\text{Energy of X-ray} = \frac{hc}{\lambda} = \frac{6.63\times10^{-34} \times 3\times10^8}{10\times10^{-9}} = 1.989\times10^{-17} \text{ J}$$

$$v = \frac{W}{Q} = \frac{1.989\times10^{-17}}{1.6\times10^{-19}} = 124\,\text{v} ✓✓$$

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