Physics Factsheet

www.curriculum-press.co.uk Number 275

Nuclear Size

Atoms are very small. It's sometimes said that if you were to blow up an orange to the size of the earth, each atom in the orange would be approximately the size of the original orange.

Q1:From this, calculate the approximate radius of an atom.
The Earth's radius is about 6400km.

However, the atomic nucleus is many tens of thousand times smaller than this – like a "pea in a cathedral" in the classic phrase. How, then, can we even think about measuring its radius? We need to take *indirect* measurements by bombarding it with other particles and seeing how they are affected. These are called *scattering* experiments.

The earliest of these experiments involved directing alpha particles at a gold foil. Famously, most of them were undeflected, but a small percentage of them deviated through a large angle – a very few even bounced straight back.

Q2:How do these results provide evidence for the existence of an atomic nucleus?

Q3: Why are the alpha particles repelled by the nucleus?

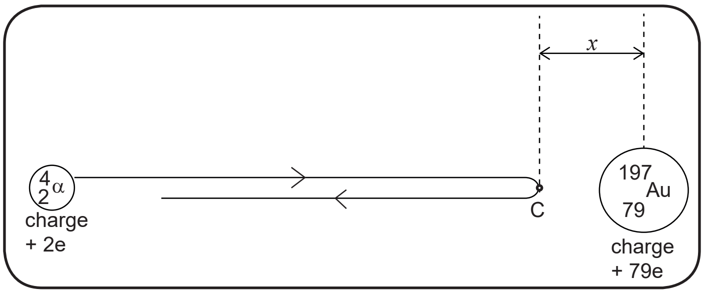
Closest Approach

Let's consider an alpha particle approaching a gold nucleus head-on. It is moving fast, with a kinetic energy of 5.0MeV.

Q4: Express the alpha particle's kinetic energy in Joules.

But as it comes closer, its kinetic energy is converted into electrical potential energy. It is doing work against the force of electrical repulsion.

The equation for electrical potential energy is $E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$

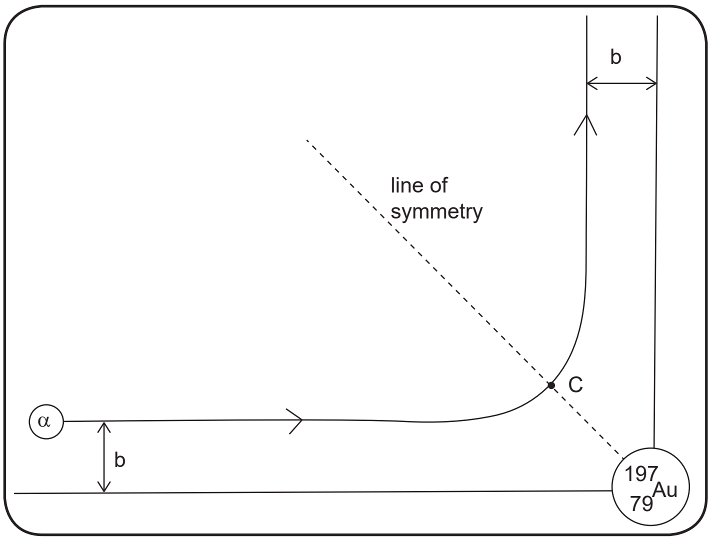


Q5:How much kinetic energy will the alpha particle have when it reaches point C, the point of closest approach? How much electrical potential energy will it have? Hence work out the distance between point C and the centre of the nucleus.

Closest approach can be used to give an upper bound on the diameter of an atomic nucleus. If the alpha particle bounces back unchanged, we can say that it failed to penetrate the nucleus. If you give the alpha particle enough energy, it will get so close that nuclear forces begin to have an effect. The alpha particle may then disrupt the nucleus.

Glancing Collision

In the diagram below, the alpha particle is *not* approaching head-on. The distance b is known as the *impact parameter.*



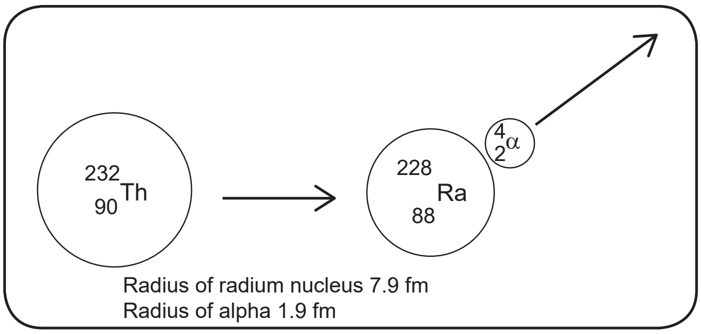
Q6:Note that the path is ***symmetrical.*** *Why?*

Q7:On the diagram, mark the direction of the force on the alpha particle when it is at position X.

Q8:Explain whether the distance of closest approach will now be ***greater, less than or the same as*** *that in Question 5.*

We can think about the reverse process, too. When a radioactive nucleus *emits* an alpha particle, it is repelled out of the nucleus at high speed. Electrical potential energy is converted to kinetic energy.

In the situation below a thorium-232 nucleus is emitting an alpha particle and transmuting into an isotope of radium. We can think of the alpha particle as being repelled by the radium nucleus and shooting out as a result.



Q9: Calculate the electrical potential energy at the beginning.

Q10:Calculate the velocity of the alpha particle at the end.

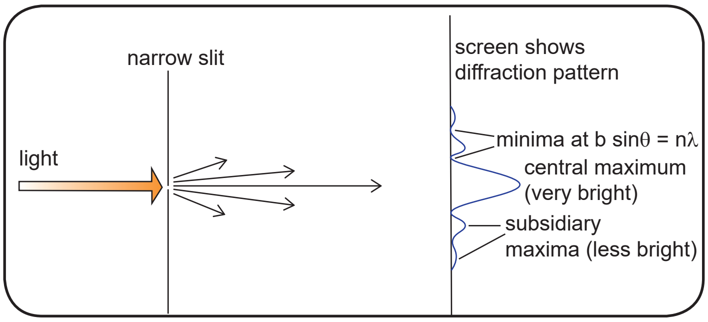
Q11:In fact the velocity of the alpha particle is considerably less than the value calculated in Q10. Why?

Electron Scattering

We have said that the "closest approach" method only enables us to puts an *upper bound* on nuclear radius. There is another technique that enables us to get a better handle on things – it is known as elastic electron scattering, and uses electron diffraction to measure nuclear size. To understand what is going on we need a firm understanding of *diffraction*.

Diffraction

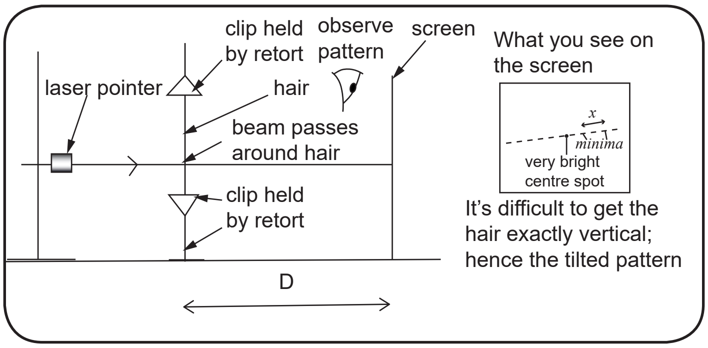
You will remember that coherent light illuminating a single slit produces a *diffraction pattern*. There is a central maximum, with the position of the first minimum given by **bsinθ= λ**



Q12:*Laser light passes through a slit of width 0.20mm. The first minimum of this pattern is observed at an angle 0.182 degrees. Calculate the wavelength of the laser.*

Babinet's Principle states that the diffraction pattern for light passing round an obstacle is *the same* as that for light passing through a gap (with the exception of theta = 0, the straight-ahead direction).

We could thus use this laser light to calculate the diameter of a hair, as in the setup below:



Q13:The laser from the previous question is now directed past a hair. The diffraction pattern on a screen 2.0 metres away has minima with gaps of 5.0cm between them. Calculate the diameter of the hair.

A similar technique can be used to calculate the diameter of the nucleus. However, we cannot use visible light for this purpose – its wavelength is much too long. We must use something with a shorter wavelength – electrons.

De Broglie Wavelength

In 1923 Prince Louis de Broglie realised that just as light waves have particle-like properties and can be thought of as photons, particles such as electrons can also be thought of as waves. The equation connecting them is **λ = h/p**, where h is Planck's constant, p is the momentum of the electron and λ is the wavelength associated with the electron.

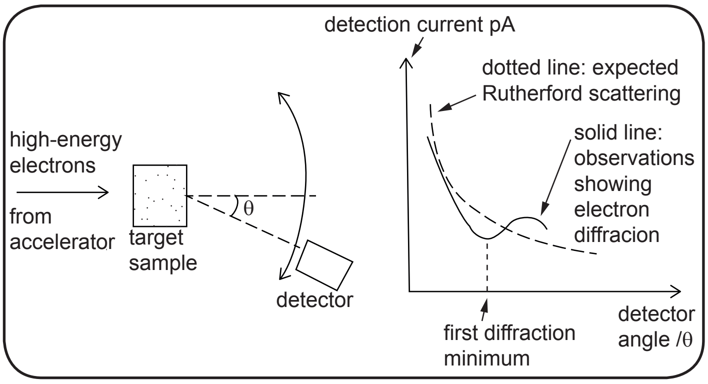
Q14:An electron is accelerated down a cathode ray tube by a potential difference of 3000V. Calculate the de Broglie wavelength of the accelerated electron.

A rule of thumb is that in order to get useful data from scattering experiments, the wavelength of the incident electrons has to be comparable to the size of the object under investigation. Since nuclei have sizes of the order of femtometres, we need electrons with wavelengths in the femtometre range, too.

This means using electrons with much higher energies and therefore, with much higher speeds. Relativistic effects, due to these very high speeds, mean that the classical equation is no use in this regime. We must use a different equation: **E= hc/λ**

Q15:How large a potential difference would be required to accelerate an electron until it had a wavelength of 5.0 femtometres?

Particle accelerators such as the Stanford Linear Accelerator (SLAC) are used to produce electrons with these energies. When these electrons are directed at nuclei, as in the experiment shown below, these nuclei act rather like the hair in the previous experiment. The electrons diffract around them, and just as there are diffraction minima of the laser light, there are diffraction minima of the scattered electrons. But because the nucleus presents a roughly circular obstacle rather than a straight-line barrier, the formula for the minimum changes to **sinθ≈1.22λ/b**, where b is the diameter of the obstacle.



The dotted line graph on the right shows the expected variation of electron intensity detected with angle if there were no diffraction (this is analogous to the Rutherford scattering of alpha particles). The solid line shows what actually happens. A diffraction minimum can be seen superimposed on the curve.

Q16: If the incident electrons had a wavelength of 5.0 fm, and the diffraction minimum occurs at 55.5□, use the equation given above to calculate the approximate diameter of the nucleus.

This could be repeated for many different electron momenta and many different nuclei. The table of data below is, alas, artificially constructed by your author - but it reflects genuine trends.

Wavelength /fm	Sin α For Mg-24	Sin α For Ca-40	Sin α For Cu-65	Sin α For Ag-107	Sin α For Au-197
2.50	0.441	0.372	0.316	0.268	0.218
2.85	0.502	0.424	0.360	0.305	0.249
3.22	0.567	0.479	0.407	0.345	0.281
3.78	0.666	0.562	0.478	0.405	0.330
4.21	0.742	0.626	0.532	0.451	0.368
4.88	0.860	0.725	0.617	0.523	0.426

Q17: For each of these nuclei, use the electron scattering data to plot a graph from which the atomic radius R can be obtained.

This "elastic electron scattering" is one of the most reliable methods of obtaining a value for nuclear radius, pioneered by Hofstader in the 1950s.Now that we have a value for R we can compare it to the mass number A for each nucleon.

Q18: Compare R^3 to A. You may wish to plot a graph. What do you notice?

We see that R^3 is proportional to A. In fact, we can generate the equation $R= r_0A^{1/3}$, with $r_0 \approx 1.2 \times 10^{-15}$m.

From this we can estimate the size of any nucleus, given its mass number.

Q19: Assuming that the nucleus is spherical, what is the relationship between its volume and the mass number? What does this imply about the nuclear density?

It appears, then, that nuclear matter is rather incompressible. Indeed, in this way it rather resembles a drop of water (although of course its density is much, much larger!). This "liquid drop" model turns out to be surprisingly powerful in predicting how atomic nuclei will behave.

On a much larger scale, a pulsar or neutron star is an example of an astrophysical object with densities comparable to that of a nucleus. It is made entirely of neutrons, although it is held together by gravity rather than by the strong nuclear force.

Q20: Estimate the radius of a neutron star with a twice the mass of the Sun (the Sun's mass is about 2.0 x 10¬30 kg)

Answers:

Q1: The orange I measured had a radius of about 4cm or 4×10^{-2} m . Earth radius is about 6400km. This would make the atomic radius about 4×10^{-2} m $\times 4 \times 10^{-2}$ m/6.4×10^6m = 0.25×10^{-10}m. This is a bit on the low side (hydrogen 0.5×10^{-10} m). Perhaps a grapefruit would be a better bet.

Q2: The deflections are *few* because the nucleus is *very small*. Most alpha particles sail by, far from the nucleus. But the fact that there are *any* indicates that the nucleus has a highly concentrated positive charge. Only in this way will the electrical field close to its centre be high enough to repel the alpha particles.

Q3: Because both the alpha particle and the nucleus are positively charged.

Q4: 5MeV is the energy an electron has when accelerated through a potential difference of 5 million volts.

It is thus equivalent to 5×10^6 V $\times 1.6 \times 10^{-19}$ C = 8.0×10^{-13} J.

Q5: At C it will have *zero* kinetic energy. *All* the kinetic energy has been converted into electrical potential energy. So, we must apply the formula to work out x.

$x=(Q_1Q_2)/(4\pi\varepsilon_0E) = 4.5 \times 10^{-14}$m or 45 fm.

Q6: This is a surprisingly difficult question. One answer might involve conservation of angular momentum. But perhaps the most intuitive is to imagine reversing the direction of time. The particle now comes in along the exit path and leaves along the entry path. Newton's laws of motion work just as well under time-reversal, so the reversed path must be just as valid as the original one. The impact parameter must thus be the same in both cases (given that a smaller impact parameter always produces a greater deflection angle).

Q7: The force arrow should point *directly away* from the nucleus.

Q8: The distance of closest approach must be slightly *greater*. The alpha particle is still moving at point X so it still has some kinetic energy left. Therefore, the electrical potential energy must be *less* than in the other case – the alpha particle is further from the nucleus.

Q9: We use the formula $E = Q_1Q_2/4\pi\varepsilon_0r$. But r is 7.9fm + 1.9fm=9.8fm, because the centre of the alpha particle is 9.8fm from the centre of the radius nucleus. This gives 4.13×10^{-12}J or 26MeV

Q10: The mass of the alpha particle is about 4 amu (1 amu is about 1.66×10^{-27} kg).

Using ½ mv^2 for the kinetic energy, we rearrange to get v = 3.5×10^7 m/s.

Q11: The alpha particle starts very close to the nucleus and thus will experience attraction due to the strong nuclear force. Work must be done against this force as it leaves the nucleus. The final energy of the alpha particle will thus be considerably reduced – it is more like 5 MeV in fact.

Q12: Using bsin θ= λ we obtain a wavelength of 635nm, typical of a red-orange laser pointer.

Q13: Using x/D ≈ λ/b and rearranging for b gives about 25μm. This experiment is easy to set up with an ordinary red laser pointer and is well worth trying. Measure the hair with a micrometer screw gauge and compare your results. A cotton thread also works well.

Q14: Using the de Broglie equation this yields 2.24×10^{-11}m – useful for looking at crystal spacings but far too big for nuclear work.

Q15: Now we have E=hc/λ which yields $E=3.987\times10^{-11}$J = 2.48×10^8 eV or 0.25 GeV. This would require 250 million volts. This is why high-energy accelerators such as the Stanford Linear Accelerator (SLAC) were used for these experiments.

Q16: Rearrange sinθ≈1.22λ/b to get b≈1.22λ/sinθ. Since λ = 5.0×10^{-15}m and θ is 55.5□, b ≈ 7.4 x 10^{15}m.

Q17: Since sin θ = 1.22 λ / b, a graph with λ on the x-axis and sin θ on the y-axis should give a gradient of 1.22 / b. Take the reciprocal of this and multiply by 1.22 to get the diameter and halve for the radius.

Q18: We should find that the cube of the radius is proportional to A, the mass number – if you plot R^3 versus A, you should get a straight line through the origin.

Q19: This implies that the volume is proportional to A as well. Since A is the mass number, we can say that volume is proportional to mass – i.e. the nuclear density is constant.

Q20: Pick one of the nuclei in Q16/17 and convert its mass to kilograms (multiply the mass number by 1.6605 x 10-27 kg, which is 1 amu). Find the ratio of the mass of the neutron star to the mass of the nucleus. The cube root of this ratio should be the ratio of the diameters. Multiply the diameter of the nucleus by this ratio to get a diameter of about 25km or a radius of about 13km, the size of a city.

N.B. this is only an order of magnitude estimate – the density of neutron stars is more variable than that of a nucleus and they are typically somewhat bigger than this (the size of a large city).

Acknowledgements:** This **Physics Factsheet** was researched and written by **Jeremy Douglas** and published in **January 2018** by **Curriculum Press**.* ***ISSN 1351-5136