PhysicsFactsheet

www.curriculum-press.co.uk Number 273

# Key Preparation: Mathematical Techniques

## Introduction

Although there are no specific new mathematical techniques which are an absolute requirement for your A-Level Physics course, you need to be secure on the techniques you learnt for GCSE Maths, and be able to apply them confidently in the context of A-Level Physics concepts. Those of you who are doing A-Level Mathematics, will be learning Calculus, but whilst the application of these techniques can be an elegant way to answer some questions, there will be an alternative approach based on GCSE Mathematics alone, so this Factsheet will concentrate on consolidating the GCSE techniques. The Factsheet does not deal with techniques of graph plotting and interpretation, nor with handling uncertainties, since these are dealt with in the Practical Factsheets.

***Task:*** *Make a list of any of the techniques you can think of: which you learnt in GCSE Maths. E.g:*
*Changing the subject of an equation.*
*Decimal and standard form.*

## Units and standard form

You should be familiar with units expressed, for example, as Nm for Newton metres, as $ms^{-1}$ for metres per second, or $Nm^{-2}$ for Newtons per square metre.

You will be expected to be familiar with the prefixes:
p (pico $10^{-12}$), n (nano $10^{-9}$), μ (micro $10^{-6}$), m(milli $10^{-3}$), c (centi $10^{-2}$), K (kilo $10^{3}$), M (Mega $10^{6}$), G (Giga $10^{9}$) and T (Tera $10^{12}$) and be able to convert accurately between them.

### Test your understanding

1 (a) A capacitor is rated as 4pF (picofarads), express this in farads in standard form.
(b) The Earth´s magnetic field at a point is given as 43μTesla, express this in standard form in T.
(c) A milliammeter reads a current as 62mA, what is this in A?
(d) The Large Hadron Collider has collision energies of 13TeV (eV is an electron volt), what is this in eV?

***Task:*** *Make sure you are familiar with how your calculator handles standard form. Many students end up an order of magnitude out, because they are not familiar with their calculators.*

## Changing the subject of an equation

Most numerical questions on an A Level paper will require you to change the subject of an equation and/or insert numerical values into an equation.

### Worked example:

The formula sheet gives $F = -GMm/r^2$ for gravitational force. Change the subject of the equation to give an expression for r.

$$Fr^2 = -GMm$$
$$r^2 = GMm/F$$
$$r = \sqrt{GMm/F)}$$

Note: the minus sign has been dropped, because there cannot be a square root of a negative number, and this minus sign simply indicates that the force is an attractive force.

### Test your understanding:

2. (a) An equation of motion is given as: $v = u + at$
Rearrange the equation to make u the subject.
(b) Rearrange $v^2 = u^2 + 2as$ to make s the subject.
(c) Rearrange: $s = ut + \frac{1}{2} at^2$ to make a the subject.

3. *The equation for the combined resistance of 2 resistors in parallel is:* $1/R_t = 1/R_1 + 1/R_2$
*Rearrange the equation to make* $R_1$ *the subject.*

4. *Rearrange* $\rho_0 = 3H_0^2/8\pi G$ *to make G the subject.*

5. *Rearrange* $V = \pi r^2 h$
(a) *to make h the subject*
(b) *to make r the subject*

6. A student incorrectly ascribes $\Omega m^{-1}$ as the unit for resistivity. Use the equation for resistivity $\mathbf{R = \rho l/A}$, where R is the resistance, ρ the resistivity, l the length, and A the cross-sectional area of the wire to show what the unit should be.

### Worked example:

An electric heater operates on 240V, if the current is 4.17A, what is its power?

$$P = V \times I$$
$$P = 240 \times 4.17 = 1000 = 1KW$$

***Exam Hint:*** *It is worth checking your Specification to familiarise yourself with which formulae, equations and data are given on the formula sheet, and which you are required to learn.*

**Test your understanding:**

7. An electric heater is rated as 2KW. If it operates on mains 240V, what current will it take?
8. (a) The output voltage of a potential divider is given by:

$$V_{out} = \frac{R_1}{(R_1 + R_2)} \times V_{in}$$

Calculate the output voltage if $V_{in}$ is 9V, $R_1$ is 10KΩ and $R_2$ is 5.6KΩ.

(b) If $R_1$ is 2.2KΩ and the input voltage is 12V, what value must $R_2$ be to give an output voltage of 5.5V?

9. Calculate the curved surface area of a cylinder of height 3m and radius 5.6cm. (SA = 2πrh).
10. A sphere has volume 17.2cm$^3$. What is its radius? (V = 4/3 πr$^3$).

## Geometry and Trigonometry

A number of contexts in A Level Physics require a competence with concepts of geometry and trigonometry.

## Surface area and volume

You should be able to calculate the areas of triangles, and areas and circumferences of circles, surface areas and volumes of regular blocks, cylinders and spheres.

**Test your understanding:**

11. (a) The surface area of a sphere is given by 4πr$^2$. If the mean distance of the Earth from the Sun is approx. 1.496 × 10$^{11}$m, calculate the surface area of the sphere on which the Earth is orbiting.

(b) The Sun´s luminosity (the total energy output per second) is 3.846 × 10$^{26}$W. Assuming no losses, calculate the radiant energy falling on a solar panel of surface area 6m$^2$ in 3 hours of sunlight.

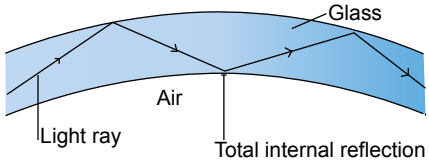
12. A section of drainage pipe is a cylinder of exterior radius 0.5m, and interior radius 0.475m and length 2.5m.
    (a) Calculate the external volume.
    (b) Calculate the internal volume.
    (c) If it is made of concrete which has a density of 2400 kgm$^{-3}$, calculate its weight.
13. Calculate a value for the mass of the Earth, given it has a mean radius of 6378Km and a mean density of 5.5gcm$^{-3}$.

## Trigonometric Ratios

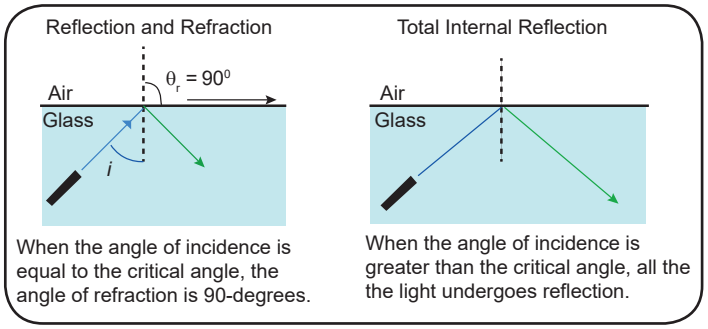
You will be expected to be able to use the ratios: sin, cos and tan, and be familiar with Pythagoras´ Theorem.

## Worked example:

A fibre optic cable uses the concept of "Total Internal Reflection" to allow light, which travels in a straight line to be transmitted along a curved cable




For TIR to occur the angle of incidence must be such that the angle of refraction is over 90$^o$ – i.e. greater than the *critical angle*. The *critical angle* is the angle of incidence for which the angle of refraction is 90$^o$.




## Practice Questions

(a) If the refractive index of the glass is 1.5, calculate the critical angle.

(b) If the glass is replaced with another material, which gives a critical angle of 60$^o$, what is its refractive index?

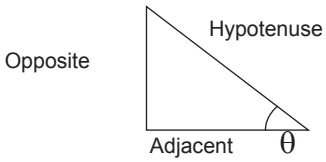
## Answers

(a) *1/ Refractive index = sin i / sin r*
*For critical angle r = 90$^o$, sin r = 1*
*sin angle c = 1/1.5 = 0.66*
*angle c = 41$^0$*

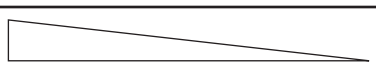
(b) *Refractive index = 1/ sin 60$^o$ = 1/ 0.866 = 1.15*

## Small angle approximation and radians

You are no doubt familiar with the ratios for sin, cos and tan:




sin θ = opposite/hypotenuse,
cos θ = adjacent/hypotenuse,
tan θ = opposite/adjacent,
but if you consider a small angle, you should be able to see that the difference in length between hypotenuse and adjacent becomes very small.



In fact, they could be considered as being equal and the radii of a circle, where the opposite is an arc. So sin θ and tan θ become the same as arc/radius. This gives the definition of an angle in radians. You will find that in A-Level Physics it is often easier to consider angles in radians than in degrees. So for small angles:

$$\sin\theta \sim \tan\theta \sim \theta \text{ rads.}$$

*For small angles: sin θ ~ tan θ ~ θ rad*

## Task:

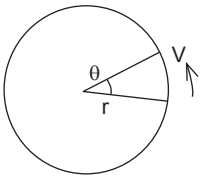
Use your calculator or tables to find sin θ and θ in radians for small angles from 0$^o$. Make a table and determine how small is "small" by finding the angle for which they differ by more than the 4$^{th}$ decimal place.

An angle of 360$^o$ is a full circle and for a circle the arc length is the circumference i.e. 2π r, so arc/ radius will be 2π

Thus 360$^o$ is 2π radians, 180$^o$ is π radians, and 90$^o$ is π/2 radians.

***Exam Hint:*** *You will find it helpful to be very familiar with these particular angles in radians. Particularly in the topics of circular motion, electromagnetic induction, interference and angular momentum.*

**Test your understanding:**

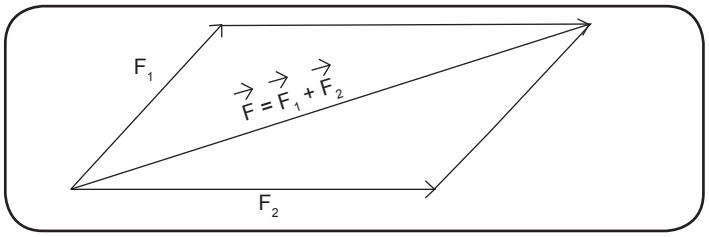



14.
(a) An object is rotating with a speed along the circumference of the circle of v $ms^{-1}$ . If the radius of the circle is r and the "angular velocity, ω " is defined as the rate at which the angle θ is increasing in radians per s (rad $s^{-1}$).
Show that: $v = r\omega$
(b) Calculate the speed at which a point on the equator of the Earth is moving through space due to the Earth´s rotation. (Take Earth´s radius as 6378km).

## Combining and Resolving vectors

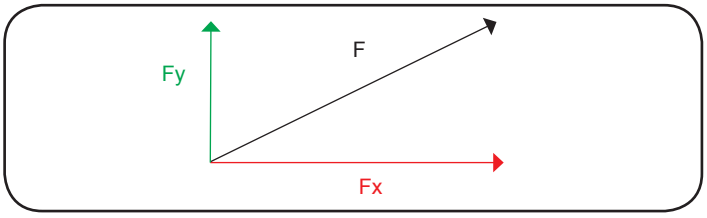
Both force and velocity are **vectors** i.e. the direction matters as well as the magnitude. Vector quantities cannot just be added or subtracted from each other like **scalar** quantities.

If 2 or more forces act on an object then the behaviour is as if it were just one force called the resultant force.



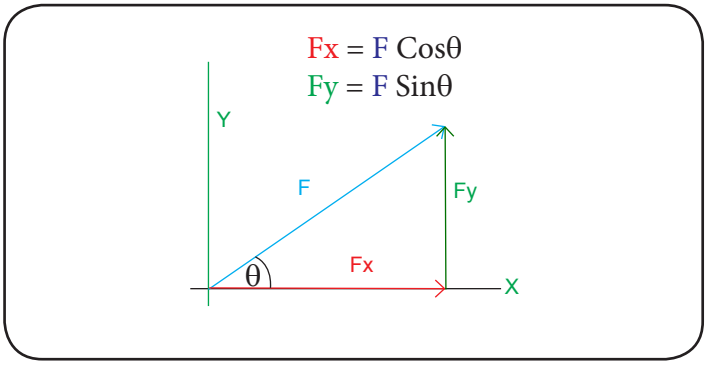

The resultant force can be found graphically as shown here or by calculation using trigonometry, which would involve solution by the triangle formula.

Equally, any force acting on an object can be considered to be the effect of two or more forces acting on it. The most useful forces are two forces at right angles and these are called the **components of the force.**



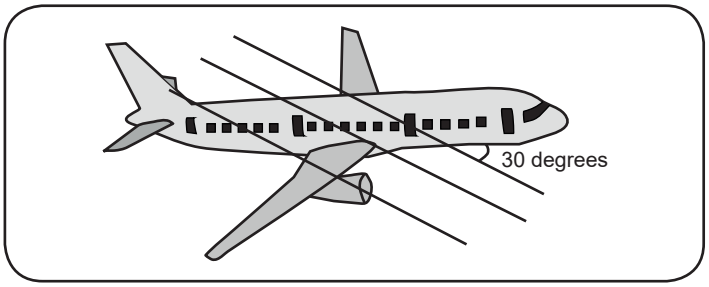

It is usual to call the two components the x- and y- components of the force.

Again the components can be obtained by scale drawing or by applying Pythagoras' theorem.




## Worked example:

A plane, with wingspan 30m, is flying straight and level at 50$ms^{-1}$ through the Earth´s magnetic field at a place where the field strength is 35 μT and the angle of dip $30^0$.




(a) Calculate the vertical component of the Earth´s field.
(b) Calculate the e.m.f. induced between the wingtips, given by the rate of change of flux linking with the wings.

**Answers**

(a) The vertical component is $35 \times \sin 30\mu T = 35 \times 0.5 = 17.5\mu T$
(b) Each second the plane´s wings map out an area of $30 \times 50$ $m^2$, so the change of flux linking with the wings each second is: $30 \times 50 \times 17.5 \times 10^{-6}$, so induced e.m.f. = 26mV.

**Test your understanding:**

15. Explain with the help of a diagram why a boat moving at a velocity of 5$ms^{-1}$ across a river with a current of 3$ms^{-1}$ must traverse at an angle of about $36.8^0$ in order to arrive at the opposite bank directly opposite where he started.

**Answers to Test your Understanding:**

1. (a) $4pF = 4 \times 10^{-12}$ F
(b) $43\ \mu T = 43 \times 10^{-6}$ T $= 4.3 \times 10^{-5}$ T
(c) $62mA = 62 \times 10^{-3}$ A $= 6.2 \times 10^{-2}$ A
(d) 13 TeV $= 13 \times 10^{12}$ eV $= 1.3 \times 10^{13}$eV

2. (a) $v = u + at$ $u = v - at$
(b) $v^2 = u^2 + 2as$ $v^2 - u^2 = 2as$ $s = (v^2 - u^2)/ 2a$
(c) $s = ut + \frac{1}{2} at^2$ $\frac{1}{2} at^2 = s - ut$ $a = 2(s - ut)/ t^2$

3. $1/R_t = 1/R_1 + 1/R_2$
Multiply by $R_t R_1 R_2$ $1R_1R_2 = 1R_t R_2 + 1R_t R_1$
$R_1(R_2 - R_t) = R_tR_2$ $R_1 = R_tR_2 /(R_2 - R_t)$

4. $\rho_0 = 3H_0^2 /8\pi G$ $8\pi G\rho_0 = 3H_0^2$ $G = 3H_0^2/8\pi\rho_0$

5. (a) $V = \pi r^2 h$ $h = V/ \pi r^2$
(b) $r^2 = V / \pi h$ $r = \sqrt{V/\pi h}$

6. $R = \rho l/A$ units: R is Ω l is m A is $m^2$
$\rho = R A /l$ is $\Omega m^2 /m = \Omega m$ not $\Omega m^{-1}$

**Answers to Test your Understanding:**

7. $P = V \times I \quad 2000 = 240\ I \quad I = 2000/240 = 8.33A$

8. (a) $V_{out} = 10000 \times 9/ (10000 + 5600) = 10000 \times 9/ 15600 = 5.77V$
   (b) $V_{out}\ (R_1 + R_2) = R_1\ V_{in} \quad 2200 + R_2 = (2200 \times 12 / 5.5) - 2200 = 2.6K\Omega$

9. S.A. $= 2\ \pi\ rh = 2\pi \times 0.056 \times 3 = 1.06m^2$

10. $V = 4/3 \times \pi\ r^3 \quad r^3 = 3V/ 4\pi = 4.11 \qquad r = 1.60cm$

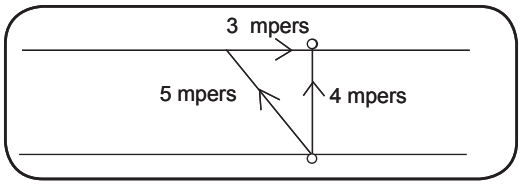
11. (a) Surface area of a sphere $= 4\pi r^2 = 4\pi \times (1.496 \times 10^{11})^2 = 28.12 \times 10^{23}\ m^2$
    (b) The total radiation is spread over this area, so amount falling on $6m^2$ per second
    $= 3.846 \times 10^{26} \times 6/28.12 \times 10^{23} = 821\ Js^{-1}$
    So in 3 hours $821 \times 60 \times 60 \times 3$ J will fall = 8.87MJ

12. (a) $V_{ex} = \pi r^2 l = \pi \times 0.5^2 \times 2.5 = 1.96m^3\ V_{in}$
    $= \pi \times 0.475^2 \times 2.5 = 1.77m^3$
    So volume of concrete = $1.96 – 1.77 = 0.19m^3$
    Mass concrete $= 0.19 \times 2400 = 456kg.$
    Weight $= 456 \times 9.81$ N $= 4473N$

13. $V_{earth} = 4/3 \times \pi \times r^3 = 4 \times \pi \times (6.378 \times 10^5)^3/3 = 1.087 \times 10^{27}\ cm^3$
    So mass $= 1.087 \times 10^{27} \times 5.5\ g = 5.98 \times 10^{27}\ g = 5.98 \times 10^{24}\ kg$

14. (a) Circum. of circle is $2\pi r$. Travelling at v $ms^{-1}$ this takes $2\pi r/v$ s. In this time $2\pi$ radians have been described so rads. per s
    $= 2\pi / (2\pi r/v) = v/r$ so $v = r\ \omega$
    (b) Earth rotates on its axis once in 24 hours, so $\omega = 2\pi/(24 \times 60 \times 60) = 7.27 \times 10^{-5}\ rads^{-1}$
    $v = 7.27 \times 6378 \times 10^{-2} = 463.7ms^{-1}$

15. In order for the resultant path to be at right angles to the river, the boat must head upstream so that the triangle of vectors is a right-angled triangle with sides 3, 4, 5 as shown.




So tan angle = ¾ = 0.75, angle = $36.8^0$

***Acknowledgements:*** *This* ***Physics Factsheet*** *was researched and written by* ***Janice Jones*** *and published in* ***January 2018*** *by* ***Curriculum Press****.*  ***ISSN 1351-5136***